# TRIAL SOLUTIONS

Suppose that $P(D)$ is a linear differential operator with constant coefficients. How to solve $P(D)y = F(x)$?

(1) Find an annihilator $A(D)$, a linear differential operator with constant coefficients, such that $A(D)F(x) = 0$.

(2) Solve $A(D)P(D)y = 0$ and the solution is of the form

$$y = y_T + y_c,$$

where $y_c$ is the general solution to $P(D)y = 0$ and $y_T$ is called a trial solution (this is a function with some undetermined constant(s)).

(3) Plug $y_T$ into $P(D)y = F(x)$ to fix the unknown constant(s).

(4) Once we fix the unknown constant(s) in $y_T$, plug the value of the constant(s) into $y_T$. Then $y_T$ becomes a fix function $y_p$. The general solution to $P(D)y = F(x)$ is

$$y = y_p + y_c.$$

How to find an annihilator $A(D)$?

(1) If $F(x) = (a_k x^k + \cdots + a_1 x + a_0)e^{ax}$, then

$$A(D) = (D - a)^{k+1}.$$

(2) If $F(x) = (c_k x^k + \cdots + c_1 x + c_0)e^{ax} \cos bx + (d_k x^k + \cdots + d_1 x + d_0)e^{ax} \sin bx$, then

$$A(D) = (D^2 - 2aD + a^2 + b^2)^{k+1} = (D - z)^{k+1}(D - \bar{z})^{k+1}, \quad z = a + ib.$$

(3) If $F(x) = F_1(x) + F_2(x)$ and $A_i(D)F_i(x) = 0$ for $i = 1, 2$, then

$$A_1(D)A_2(D)F(x) = 0$$